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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 1, 2019/2020

**TMT 1181 – MATHEMATICAL TECHNIQUES II**

(All sections / Groups)

14 OCTOBER 2019

2.30 p.m. – 4.30 p.m.

(2 Hours)

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### INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **6 pages** only including the cover page with **5 Questions** and an **Appendix**.
2. Attempt **ALL FIVE (5)** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page**.
4. Show all steps.

Answer ALL FIVE (5) Questions.

**Question 1**

a. Identify the *linearity* and *order* of the following differential equations:

i.  $2\frac{d^3y}{dx^3} + 5y = 3$  [1 mark]

ii.  $-y'' - 8y' + y = 5t$  [1 mark]

b. A *first order differential equation* (DE) is given as below:

$$\frac{dy}{dx} = \frac{4x^3}{y^2 + 6y}$$

i. Show that the DE is *separable*. [1 mark]

ii. Solve the DE using *separable method*. [3 marks]

c. A *first order differential equation* (DE) is given as below:

$$9y' + 9y = 9x$$

i. Rewrite the DE into the form of  $y' + p(x)y = r(x)$ . Identify  $p(x)$  and  $r(x)$ . [2 marks]

ii. Find the *integrating factor*,  $u(x) = e^{\int p(x)dx}$ . [2 marks]

Continued.....

**Question 2**

- a. Given that the *roots of the characteristic equation* for the following *differential equation* (DE)

$$ay'' + by' + cy = 0$$

are  $\lambda_1 = -5$  and  $\lambda_2 = -5$ ,

- i. find the values of  $a$ ,  $b$  and  $c$ . [2.5 marks]
  - ii. write down the *general solution*,  $y$ , of the DE. [0.5 mark]
- b. Given the following DE:  $y'' - 5y' + 4y = 5x$ ,
- i. determine whether the DE is *homogeneous* or *non-homogeneous*. If it is non-homogeneous, convert it to homogeneous DE. Then solve it to get  $y_h$ . [3 marks]
  - ii. find the *particular solution*,  $y_p$ , for the DE using *undetermined coefficients method*. [3.5 marks]
  - iii. form the *general solution*,  $y = y_h + y_p$ . [0.5 mark]

**Question 3**

- a. Determine the *Laplace transform* for the following functions:

i.  $f(x) = \cos 2x + 2x$  [2 marks]

ii.  $f(x) = 3x^3 e^{-3x}$  [2 marks]

- b. Given the DE,  $2y'' - y' + y = 6$ , show that the *Laplace transform* of  $y(t)$  is

$$Y(s) = \frac{6}{s(2s^2 - s + 1)} \text{ if } y(0) = 0 \text{ and } y'(0) = 0. \quad [3 \text{ marks}]$$

- c. The *partial fraction* of  $F(s) = \frac{5s+12}{(s+2)(s+3)}$  is given in the form of  $\frac{2}{(s+2)} + \frac{3}{(s+3)}$ . Find the *inverse Laplace transform*,  $L^{-1}[F(s)]$ . [3 marks]

Continued.....

**Question 4**

Given the function  $f(x) = \sqrt{1 + 2x}$  and let  $x_0 = 0$ ,  $x_1 = 0.4$  and  $x_2 = 0.6$ .

- a. Construct the Lagrange coefficient  $L_0(x)$ ,  $L_1(x)$  and  $L_2(x)$ . [3 marks]
- b. Find  $f(x_0)$ ,  $f(x_1)$ , and  $f(x_2)$ . [3 marks]
- c. Find the *second degree Lagrange interpolating polynomial*,  $P_2(x)$ . [2 marks]
- d. Approximate  $f(0.5)$ . [2 marks]

**Question 5**

- a. Given the following linear system of equations:

$$4x - y + z = 7$$

$$4x - 8y + z = -21$$

$$-2x + y + 5z = 15$$

- i. Write the system above in *matrix notation*. [1 mark]
  - ii. Represent the system above as an *augmented matrix*. [1 mark]
- b. Refer to the linear system of equations in (a).
- i. In what condition the system in (a) can be solved using *Gauss-Seidel method*? [1 mark]
  - ii. Solve the system using *Gauss-Seidel method* by performing the first **TWO** iterations. Given  $x_0 = 0$ ,  $y_0 = 0$  and  $z_0 = 0$ .  
[Note: You may leave your answers in fraction form or decimal numbers. However, decimal numbers must be rounded to 4 decimal places] [7 marks]

Continued.....

## APPENDIX

LAPLACE TRANSFORMS OF SOME ELEMENTARY FUNCTIONS

| Function, $f(t)$    | Laplace transform, $F(s)$       |
|---------------------|---------------------------------|
| 1                   | $\frac{1}{s}$                   |
| $t$                 | $\frac{1}{s^2}$                 |
| $e^{\alpha t}$      | $\frac{1}{s - \alpha}$          |
| $\sin \omega t$     | $\frac{\omega}{s^2 + \omega^2}$ |
| $\cos \omega t$     | $\frac{s}{s^2 + \omega^2}$      |
| $t^n e^{-\alpha t}$ | $\frac{n!}{(s + \alpha)^{n+1}}$ |
| $f'(t)$             | $s \cdot F(s) - f(0)$           |
| $f''(t)$            | $s^2 F(s) - s f(0) - f'(0)$     |

General solution of the homogeneous linear second order differential equation

For Distinct real roots:  $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

For Repeated roots:  $y = (c_1 + c_2 x) e^{\lambda x}$

For Complex conjugate roots:  $y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$

Continued.....

**Lagrange interpolating polynomial**

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)}f(x_0) + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)}f(x_1) \\ + \frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)\dots(x_2-x_n)}f(x_2) + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})}f(x_n)$$

**Or**

$$f(x) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2) + \dots + L_n f(x_n)$$

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